

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1371

APPARATUS FOR MEASUREMENTS OF TIME AND SPACE CORRELATION

By A. Favre, J. Gaviglio, and R. Dumas

Translation of "Appareil de Mesures de la Correlation Dans le Temps et
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APPARATUS FOR MEASUREMENTS OF TIME AND SPACE CORRELATION*1

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SUMMARY

These researches are made at the Laboratoire de Mécanique de l'Atmosphère de l'I.M.F.M., for the Office National d'Etudes et de Recherches Aéronautiques (O.N.E.R.A.), with the aid of the Ministère de l'Air, and of the Centre National de la Recherche Scientifique (C.N.R.S.).

We have already explained in previous communications (refs. 1 to 22) the principle and have given description of the apparatus for statistical measurements of time and space correlation between two random variables considered with a relative delay T in time, which we have designed with a special view to the study of turbulence.

We shall briefly review now the improvements brought to the experimental apparatus, the control of the measurements, and a few particular applications.

EXPERIMENTAL APPLIANCES

Wind Tunnel (Refs. 2, 3, 4, 6, 12, and 13)

The low-turbulence wind tunnel, installed indoors, has been transformed (fig. 1). The test section A is $80 \times 80 \times 270$ cm; the cone B with low gradients has a contraction ratio of 15. It is preceded by a settling chamber C, in which are set five removable damping screens, and by a rapid expansion diffuser D with a screen analogous to those of the low-turbulence wind tunnel of the R.A.E. (ref. 23). The intensity of turbulence is of the order of 0.00210 with the diffuser screen and

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of 0.00130, 0.00090, 0.00055, 0.00045, and 0.00038 with 1, 2, 3, 4, and 5 damping screens, respectively.

At the entrance of the test section may be placed one of the two turbulence grids similar to those of the NBS with elements of 0.626 inch and 0.196 inch in diameter and with meshes M of $3\frac{1}{4}$ inches and 1 inch, respectively.

A flat plate of $1,960 \times 80$ cm can be set in the test section for the study of the boundary layer.

Hot-Wire Anemometers (Refs. 2, 3, 5, 7, 8, 11, 17, and 20)

Two hot wires can be moved along the axis X_1 of the test section and along the orthogonal axis X_3 .

The amplifiers reproduce the oscillations with an approximation of ± 6 percent in energy and for the band pass 1 to 6,000 cps, more recently, for 1 to 3,500 cps with ± 4 percent. The compensation is made by the square waves method.

Figure 2 shows the two amplifiers A, the power supplies B, the square waves generator C, and a generator of sine waves from 0.6 to 30,000 cps D.

Apparatus for Statistical Measurements of Time and

Space Correlation (Refs. 1, 2, 3, 5, 8, 9, 10, 11, and 13)

The apparatus records simultaneously and reproduces with a relative delay T the two voltages coming, for instance, from the two anemometers and measures the time-correlation coefficient (refs. 1, 2, and 3).

It must be borne in mind that, for this measurement, it is necessary and sufficient that the phase shifts of the component waves due to the inevitable transformations should be identical in the two channels for each frequency; the difference T of the delays systematically introduced being the same for all the frequencies.

The voltages are recorded on magnetic tape:

- (a) directly for the waves of frequencies comprised between 200 and 2,500 cps

(b) on an amplitude modulated wave of 3,500 cps frequency, demodulated after reproduction, for the waves of frequencies comprised between 1 to 25 cps

(c) by overlapping of the two devices from 25 to 200 cps.

The band pass is therefore unbroken from 1 to 2,500 cps (refs. 8, 9, 10, and 11).

The apparatus consists of (fig. 2):

The two recording preamplifiers E, with a common oscillator of 3,500 cps, and on either channel the networks effecting the division of the respective spectra into waves of 200 to 2,500 cps for direct recording, of 1 to 25 cps for modulation of the 3,500 cps carrier, and of 25 to 200 cps for the aforementioned overlapping

The two recording amplifiers F (with appended oscillators of 80,000 cps and of 40,000 cps) and their power supplies G

The two magnetic recorders-reproducers H, one of which has a device I for adjusting the length of the tape between the recording and play-back heads and, consequently, the delay T (refs. 2 and 3)

The two play-back preamplifiers F

The two play-back amplifiers J, with their networks effecting, after the detection of the carrier, the reconstitution of the initial spectrum on each channel

The two power amplifiers K which, after adding on one hand and subtracting on the other, supply the thermocouples; the output voltages of these thermocouples represent the mean squares M_a of the sum and M_b of the difference, respectively (Dryden method); two extra amplifiers for the oscilloscope

The apparatus L, which gives the sum and the difference of M_a and M_b (and will soon reckon their quotient), together with an oscillator for 20, 200, 1,000 cps, by means of which may be effected a supplementary control of the phase and energy response of the two channels during the process of measurements; this oscillator is also used for the measurement of the delay T

The power supplies N; the oscillograph O; the galvanometer M with adjustable period (3 sec).

The equality of phases on the two channels is checked for each frequency (refs. 2 and 3).

When the two voltages correspond to two variables measured at two different points of space, one obtains the coefficient R of time and space correlation.

In the particular case when it represents the same variable measured at the same point of space, one gets the autocorrelation coefficient from which the spectral function can be deduced.

Spectral Analyser (Refs. 9, 10, 13, 14, 15, and 16)

A selective feed-back amplifier has been designed in order to determine the spectral function directly, apart from the autocorrelation method.

The band-pass is in width 4.5 percent of the median frequency chosen for frequencies comprised between 1.5 and 2,500 cps.

The energy is measured by means of a thermocouple and a long-period galvanometer (19 sec).

CONTROL OF THE MEASUREMENTS (REFS. 2 to 17, AND 21)

I. The response curve of the amplifiers of the hot-wire anemometers (ref. 7) has been determined as regards energy and phase by means of sine waves and square waves of variable frequencies, and so has the band-pass of the spectral analyser.

II. The total response curve of the apparatus for measurements of time correlation, together with the hot-wire anemometers, has been established with sine waves of variable frequencies, as regards the energy (fig. 3), the equality of phases on the two circuits for each elementary oscillation (refs. 2 and 3) (the delay T being a multiple of the period), and the autocorrelation function (refs. 2 to 15).

The energy is constant, variations not exceeding ± 16 or 19 percent of the standard deviation 0.03 for 1 to 2,500 cps (refs. 2 to 16).

Frequent checkings are made in the course of measurements especially in the case of 20, 200, and 1,000 cps oscillations frequencies.

III. The spectra are obtained on one hand by Fourier transform of the autocorrelation curves and on the other hand directly by means of the spectral analyser and are then systematically compared. These measurements are made in the case of turbulence downstream of a grid

(refs. 9, 10, 13, 14, and 24) and also in the case of a boundary layer of a flat plate (examples: figs. 4, 5, and 8) (refs. 15 and 24).

These spectra are satisfactorily concordant from 5 to 2,000 cps now (fig. 4).

IV. The vibrations of the hot-wire holders (ref. 2) and of the tunnel walls (ref. 4) have been examined and reduced; their effects are watched with the oscilloscope during the experiments.

The effect on the measurements of the acoustic resonances has been found to be negligible (refs. 6 and 12).

V. Comparative measurements have been made (refs. 4, 6, 9, 10, 12, and 13) at each stage of the transformations of the wind tunnel:

- (a) Length of the test section reduced from 400 to 270 cm, contraction ratio of the cone increased from 6.4 to 15, the new settling chamber placed behind the diffuser with its screen, and five damping screens inserted successively in the bulge
- (b) Installation of the first part of the return circuit with two corners with vanes
- (c) Size of the test section reduced from 80×80 cm to 80×40 cm (ref. 13).

These comparisons have not shown any appreciable changes in the results from one stage to the other, as regards the turbulence behind a grid.

One must except, however, the influence of the initial turbulence and, consequently, of the damping screens, which is very important in the case of the boundary layer and the shape of the grid.

VI. The comparison of the time correlation curves, obtained with different hot wires, disclosed appreciable differences for wires of the same diameter and especially for wires of different diameters. The improvement of the compensation by the square-waves method has made it possible to reduce those differences (fig. 6) (refs. 10 and 13).

Aerodynamic taring attempts of the hot-wire anemometer are in progress, the wire being placed in the field of a propeller (refs. 17 and 20).

The measurement errors seem to be mainly due to the hot wires; the comparative experiments are preferably made with practically identical wires.

VII. The results obtained for the turbulence behind a grid with the apparatus for time and space correlation measurements:

transversely, i.e. at several distances X_3 (refs. 9, 10, 16, 21, and 24)

longitudinally, i.e. at several distances X_1 (refs. 14, 16, and 21)

the delay T being zero and agreeing with the results concerning the transverse and longitudinal space correlations g and f , respectively, (example: fig. 7) (refs. 9, 10, 21, and 24).

VIII. The comparison of the spectra (fig. 8) (refs. 13 and 24) with those of H. L. Dryden (NBS) and of F. G. Simmons (NPL) established for the turbulence behind a grid in quite similar conditions is satisfactory. The measurements have been especially developed up to 1.5 cps. The comparison with A. A. Townsend's spectra (ref. 22) is also satisfactory although the experimental conditions are slightly different.

IX. The qualitative comparison of the spectra established for the turbulence of a flat-plate boundary layer (fig. 9) (ref. 15) with those of A. A. Townsend is fairly satisfactory, although the experimental conditions present some differences (ref. 25).

A FEW APPLICATIONS OF TIME CORRELATION MEASUREMENTS

Among the applications which can be made of time-correlation measurements between two variables or of autocorrelation for one variable, and, in addition, to those concerning the turbulence studied elsewhere (ref. 22), we shall mention some other experiments (refs. 18 and 19).

Application of Time Correlation to the Relative Increase of a

Periodic Signal Disturbed by a Random Parasite (refs. 3, 18, and 19)

1. In a previous paper (ref. 3) we have considered the coefficient of time correlation $R_{11,22}$ between two signals u_1 and u_2 disturbed by two parasites ϵ_1 and ϵ_2 , respectively, and the coefficient of time correlation $R_{10,20}$ between the nondisturbed signals² and noticed that, if one can choose a delay T , such as the time correlation coefficient between the parasites and between the signals and the parasites, to be negligible

²With $\overline{u_1} = \overline{u_2} = \overline{\epsilon_1} = \overline{\epsilon_2} = 0$, $q_1 = \overline{u_1^2} / \overline{\epsilon_1^2}$, $q_2 = \overline{u_2^2} / \overline{\epsilon_2^2}$.

$$\overline{u_1 \epsilon_2} = \overline{\epsilon_1 u_2} = \overline{u_1 \epsilon_1} = \overline{u_2 \epsilon_2} = \overline{\epsilon_1 \epsilon_2} = 0 \quad (1)$$

one finds that

$$R_{11,22} = R_{10,20} \sqrt{q_1 q_2} / \sqrt{(1 + q_1)(1 + q_2)}$$

The coefficient of time correlation between the disturbed signals is proportional to the coefficient of time correlation between the non-disturbed signals, when the delay is such that the coefficients of time correlation between the parasites and between the signals and the parasites are zero.

With periodic signals and random parasites, condition (1) is easily fulfilled; thus, these signals can be detected. With sine waves of the same frequency, the coefficient of time correlation $R_{11,22}$ or of autocorrelation $R_{11,11}$ will be for appropriate delays a sine wave of the same frequency, which will constitute a time detection of these signals (ref. 18).

In the case of a single signal, it may be preferable to replace the autocorrelation $R_{11,11}$ by the time correlation $R_{11,12}$ with an identical signal disturbed by a less intense parasite.

2. We have made trials of the detection of a sine wave by autocorrelation (refs. 18 and 19). For example, figure 10 (ref. 19) shows the autocorrelation curves of the parasite alone - the spectrum of which stretches mainly from 400 to 1,100 cps - and of the signal of ≈ 600 cps disturbed by the parasite.

It may be seen that, for a delay T superior to 8 ms, the latter is close to the sine wave of ~ 600 cps of amplitude 0.4.

Other tests have enabled us to verify the above formula and to detect the presence of the signal for very small values of q with a great sensitivity (ref. 19).

Reduction of Parasites Ratio With Reference to

Disturbed Phenomena by Addition With Delay (Refs. 18 and 19)

1. Let u_t be the variable representing a phenomenon disturbed by a parasite ϵ_t . Let us add the disturbed variable to itself with a time delay T ; we then have³

$$\frac{Q}{q} = \frac{1 + R_{10,10}}{1 + R_{01,01}}$$

Thus, it is possible to reduce the parasite disturbing a phenomenon by addition with delay. This reduction will be at its maximum when T is such that the ratio of the autocorrelation coefficients of the phenomenon, and of the parasite, to both of which the unity has been added, will be at its maximum.

In the case of a periodic variable u_t and of a random parasite, one may take a delay multiple of the period and such as

$$R_{01,01} \ll 0$$

then,

$$\frac{Q}{q} \gg 2$$

It may be preferable to filter the parasite so that, for a T multiple of the period, one should have

$$R_{01,01} \sim -1/2$$

then,

$$\frac{Q}{q} \sim 4$$

The operation may be repeated in series, choosing the optimum delays; one may also make several additions in parallel with optimum delays.

$$\text{}^3\text{With } \overline{\epsilon_t^2} = \overline{\epsilon_{t+\tau}^2}, \quad \overline{u_t^2} = \overline{u_{t+\tau}^2}, \quad q = \overline{u_t^2} / \overline{\epsilon_t^2},$$

$$Q = \overline{(u_t + u_{t+\tau})^2} / \overline{(\epsilon_t + \epsilon_{t+\tau})^2}.$$

2. We have made (ref. 18) some measurements, using a sine signal of frequency n and, for the parasite, the above-mentioned turbulent velocity fluctuations.

For a delay T such that

$$R_{01,01} \sim 0$$

and

$$R_{10,10} \sim 1$$

n	157	180	210	500	570	570	570
Q/q	1.90	1.97	2.04	2.41	2.04	1.91	2.22

the mean value 2.08 approximates the theoretical value 2.

For a delay T such that

$$R_{01,01} \sim -0.5$$

and

$$R_{10,10} \sim 1$$

and $n = 80$, the ratio Q/q reached 4.25, 4.52, and 4.42, slightly above the theoretical value 4.

Translated by A. Favre

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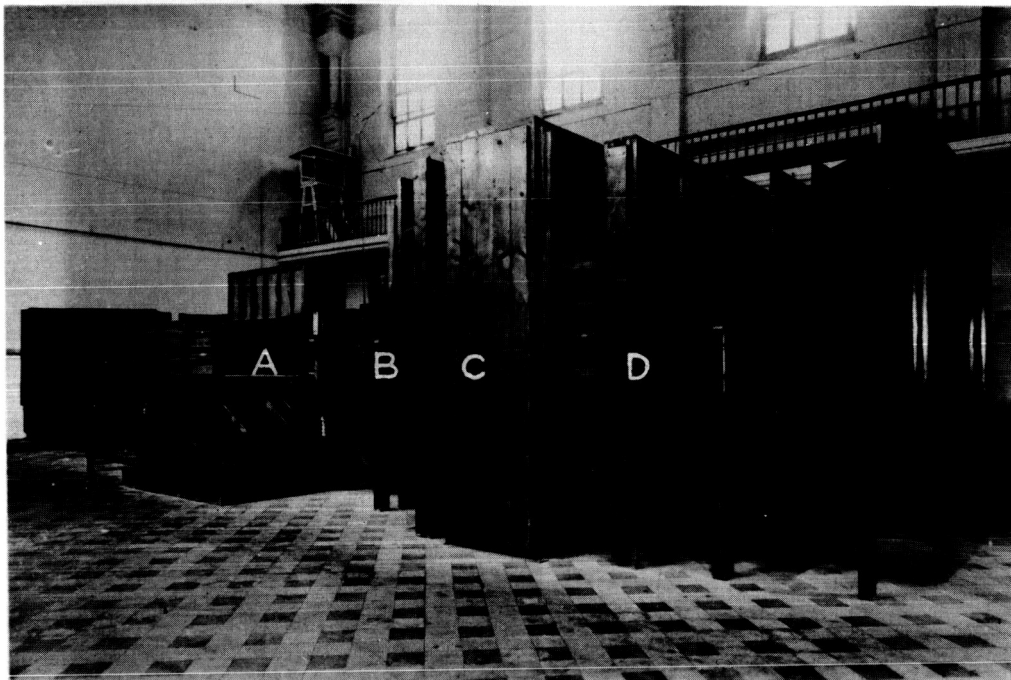


Figure 1.- Low-turbulence wind tunnel.

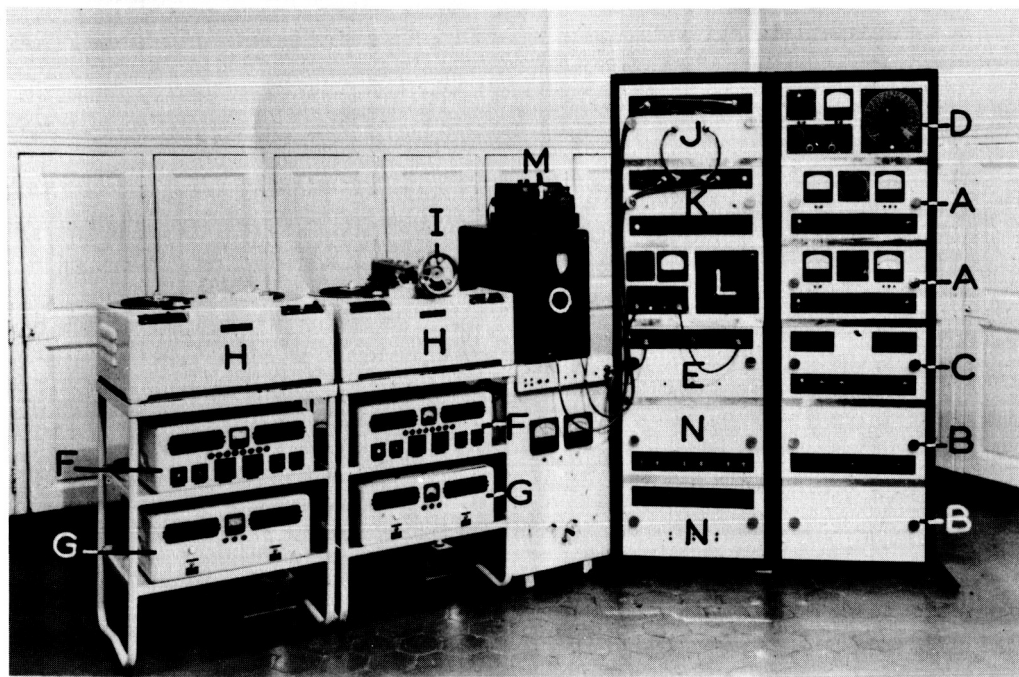


Figure 2.- Apparatus for measurements of time correlation.

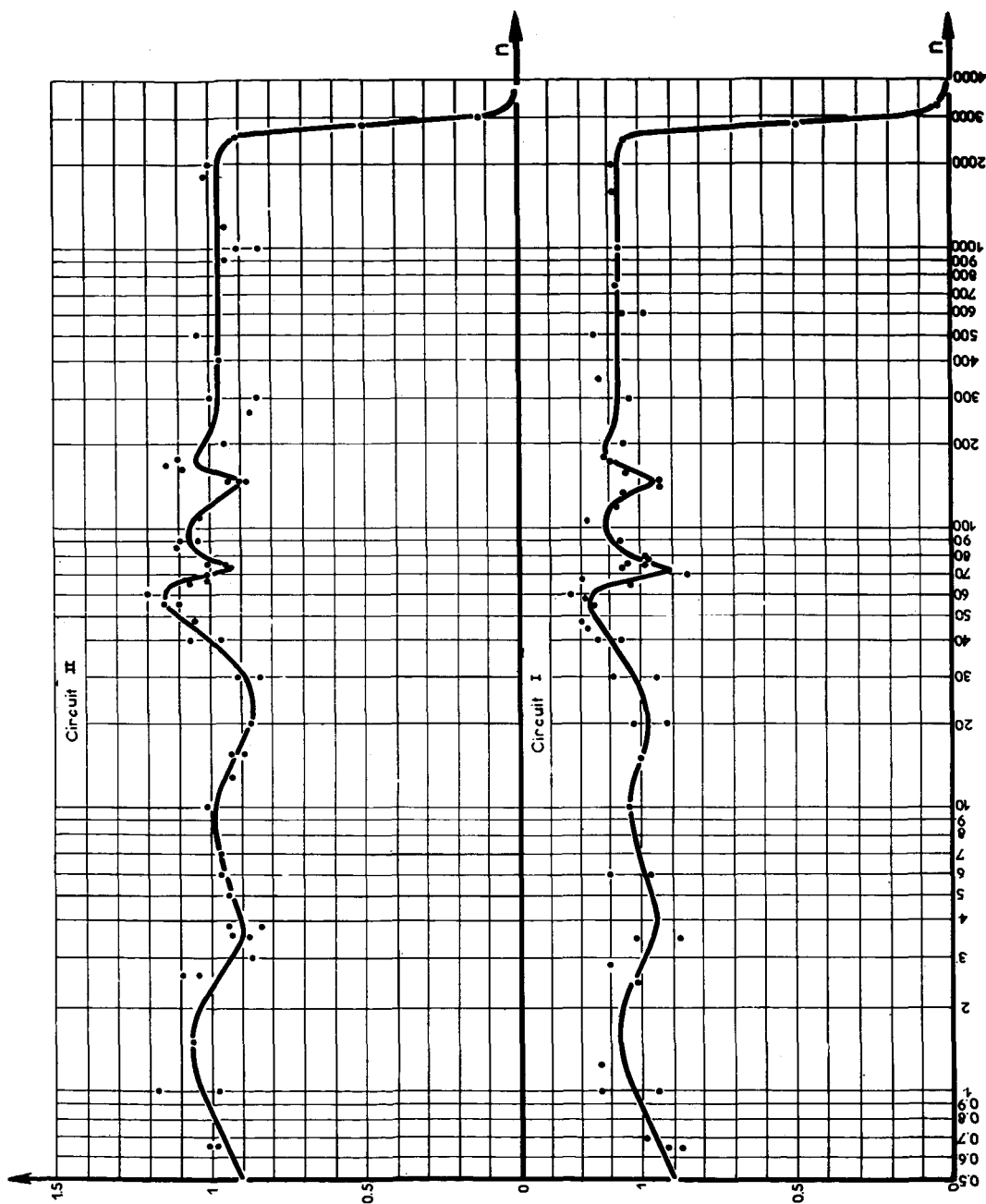


Figure 3.- Energy response of apparatus for measurements of time correlation.

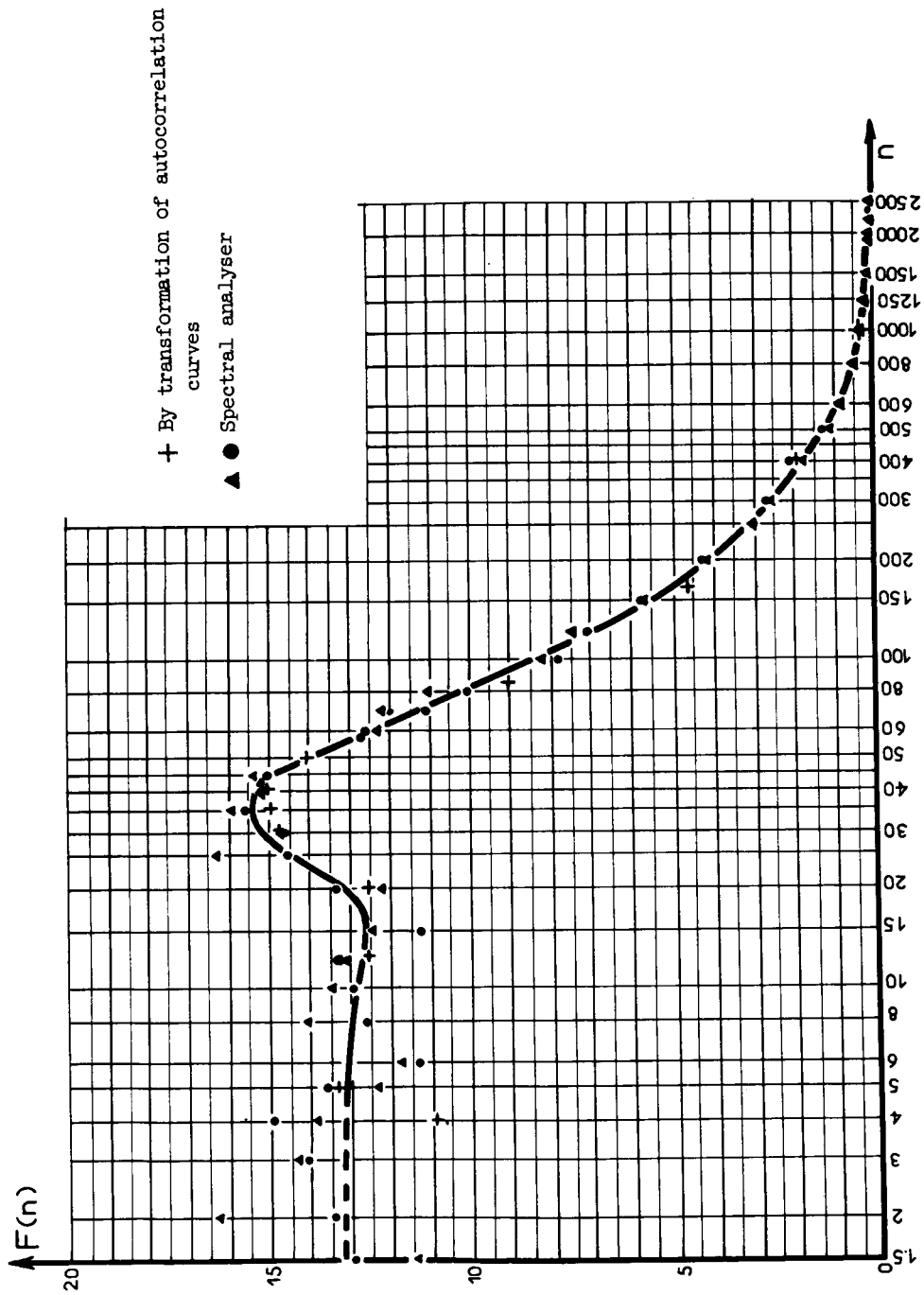


Figure 4.- An example of spectrum obtained by Fourier transform of autocorrelation curve, and spectrum directly measured; turbulence in downstream of a grid. $V = 12.27$ mps; $M = 1$ in; Distance = 40 M; $R_M = 21500$.

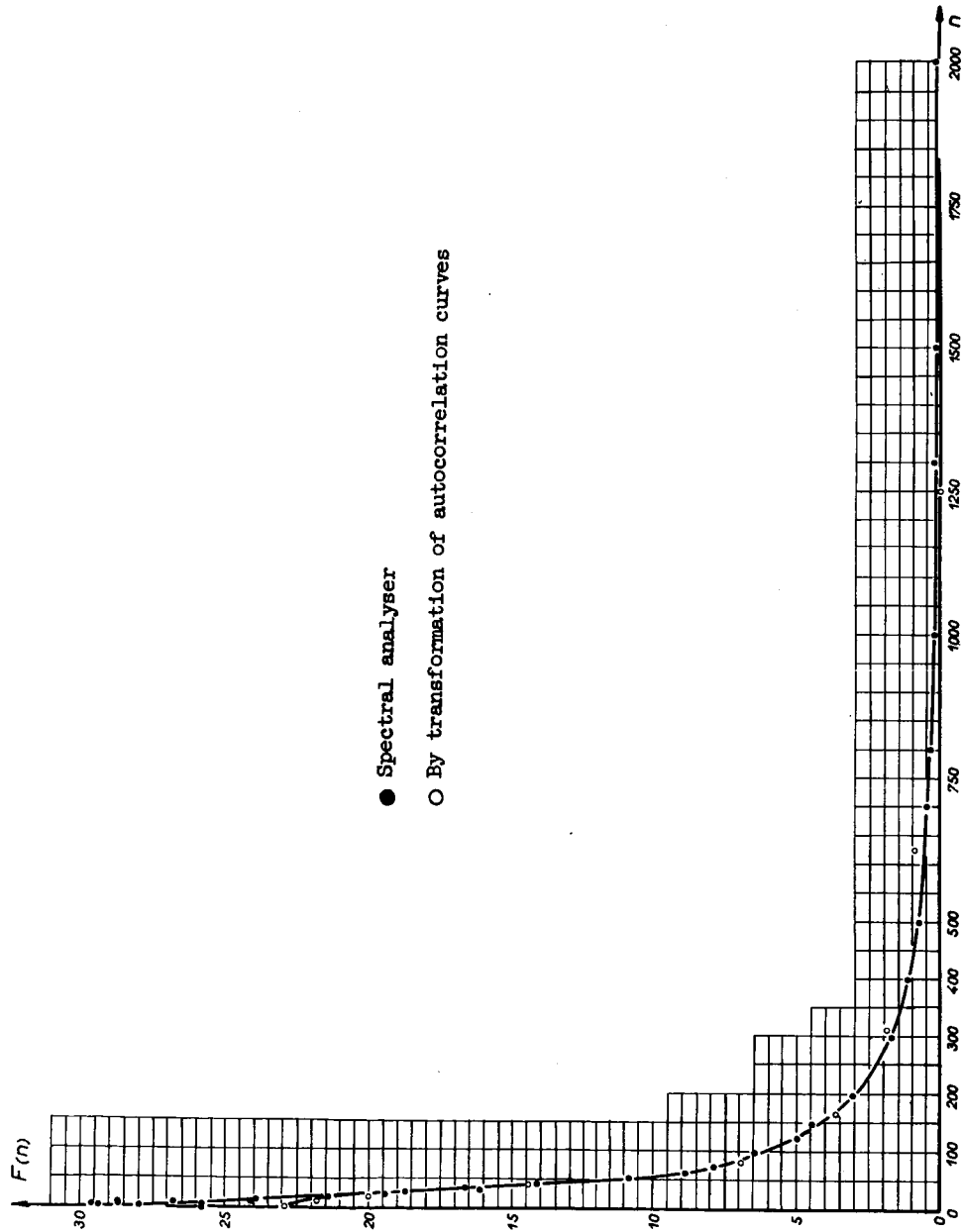


Figure 5.- An example of spectrum obtained by Fourier transform of autocorrelation curve, and spectrum directly measured; turbulence in flat-plate boundary layer. Distance relative to wall, $X_3/\delta = 0.5$; $R_{X_3} = 766000$.

Velocity = 12.20 mps; $M = 1$ in; Distance to grid = 40 M

Semiempirical compensation

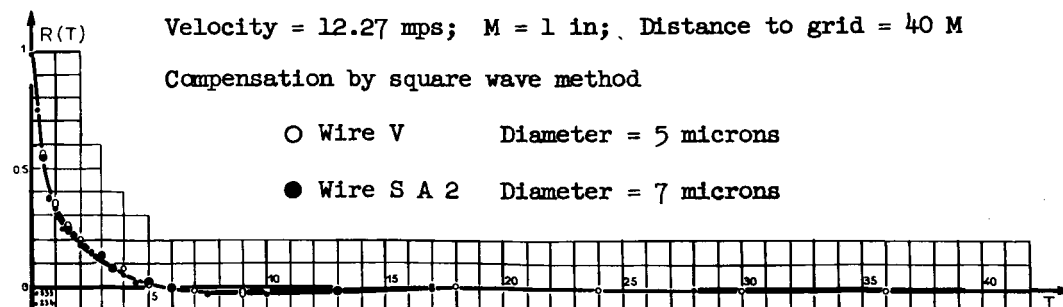
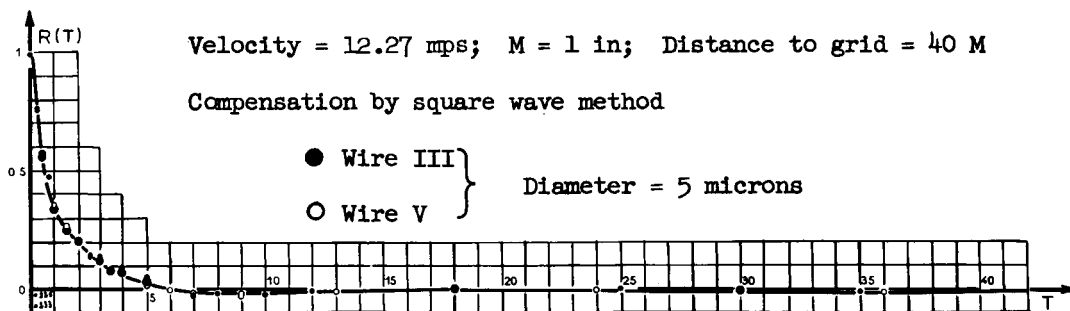
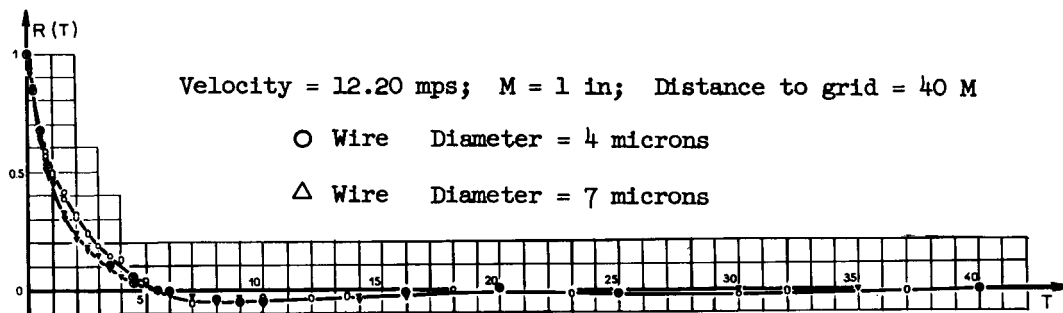
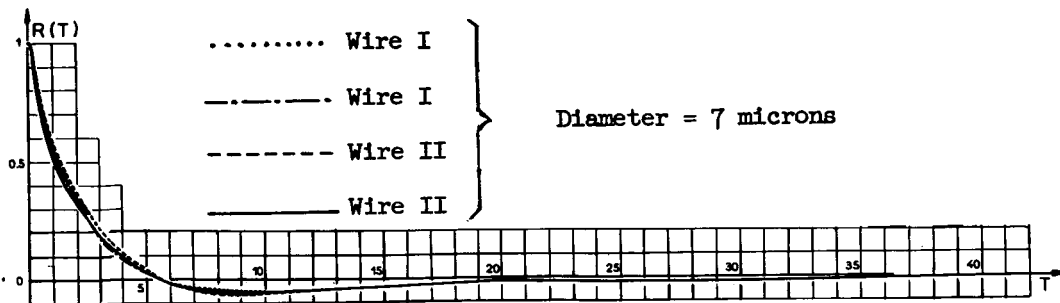


Figure 6.- Autocorrelation measured by different hot wires and by two methods of compensation.

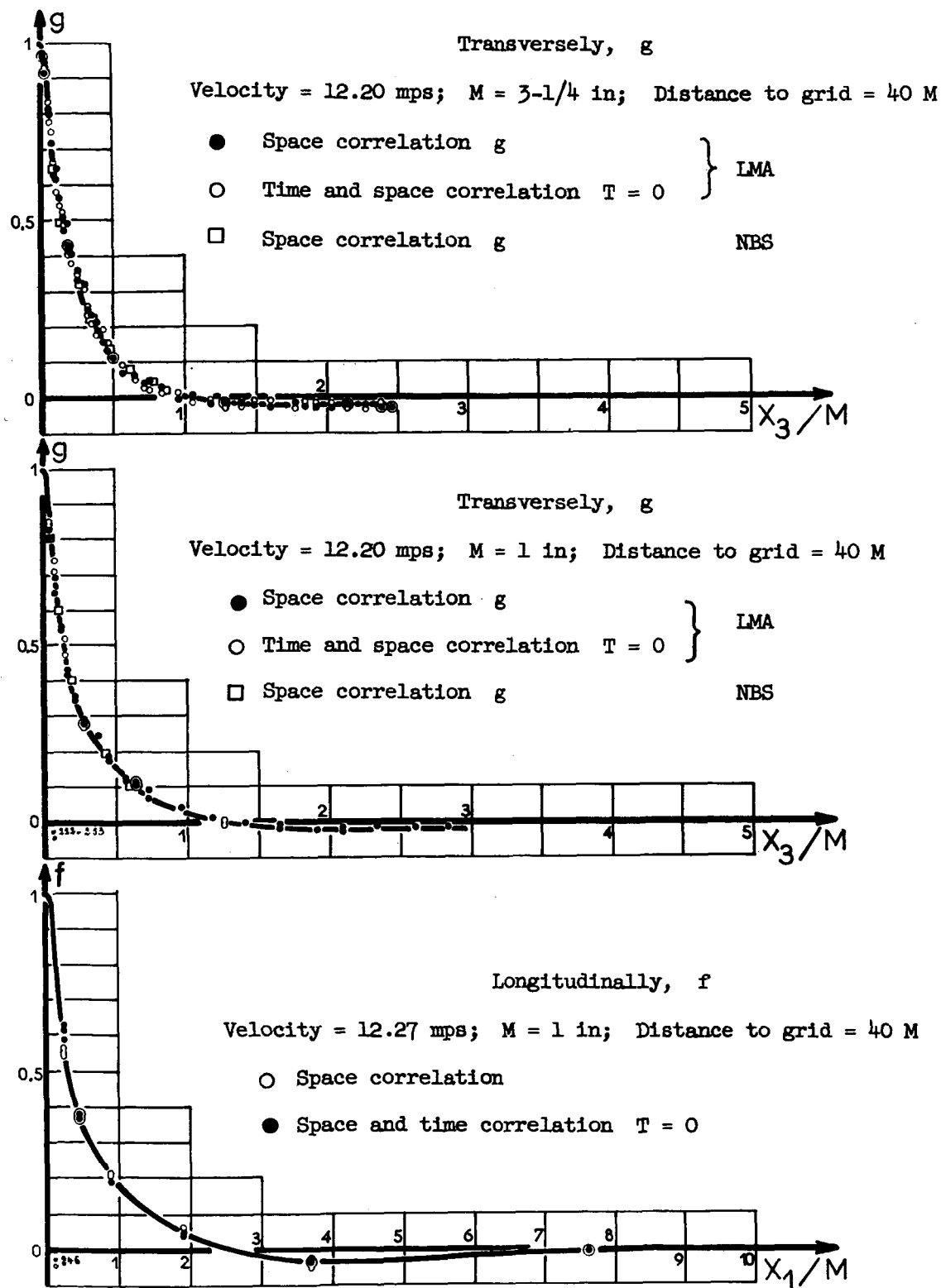


Figure 7.- Control of time and space correlation measurements with zero delay, transversely g , longitudinally f , downstream of a grid.

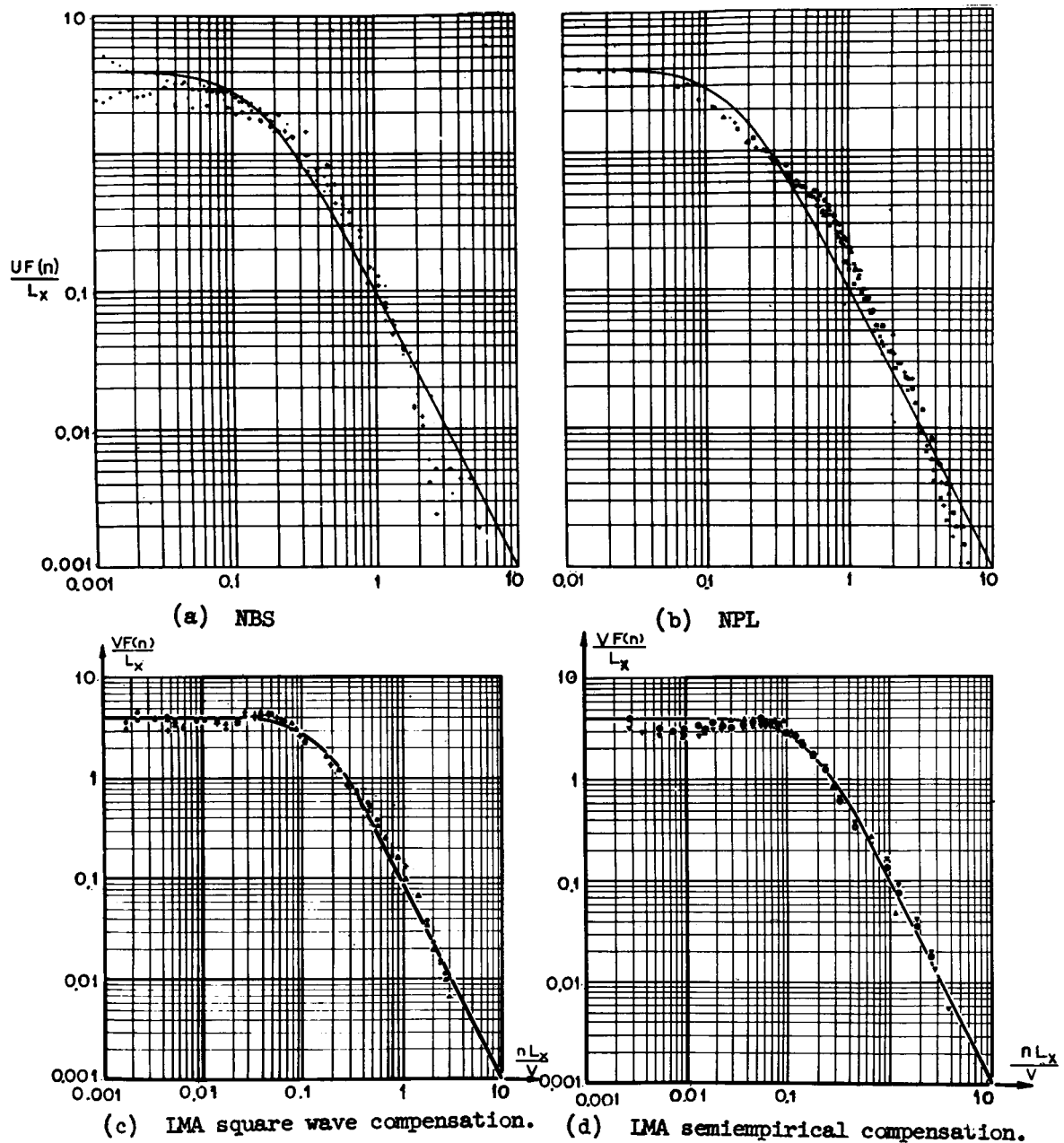


Figure 8.- Comparison of turbulence spectra downstream of a grid obtained at NBS, NPL, and LMA.

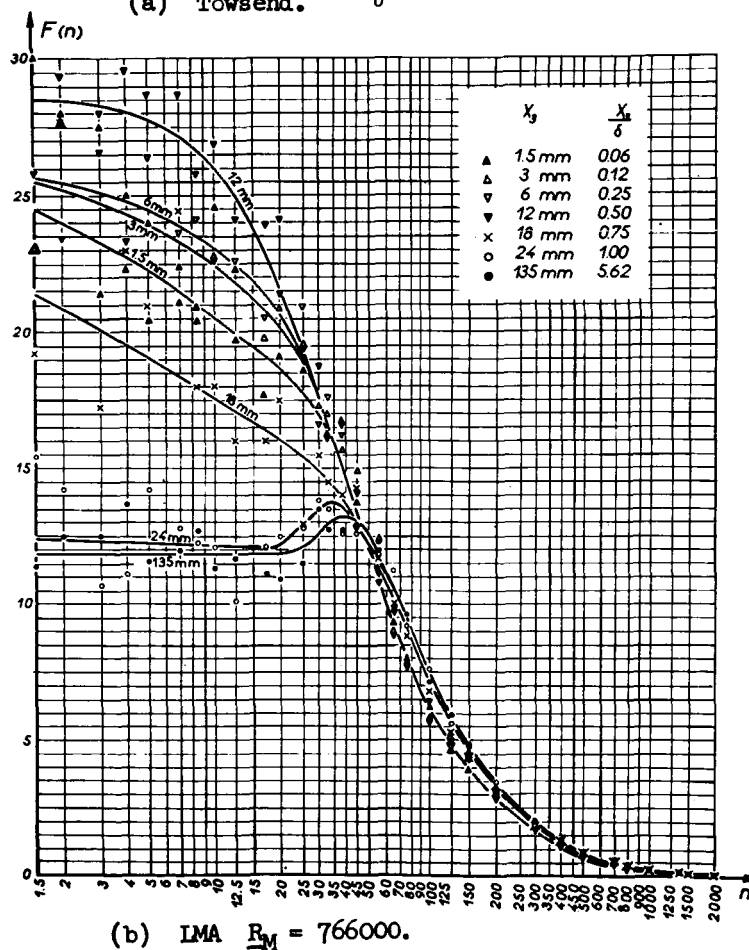
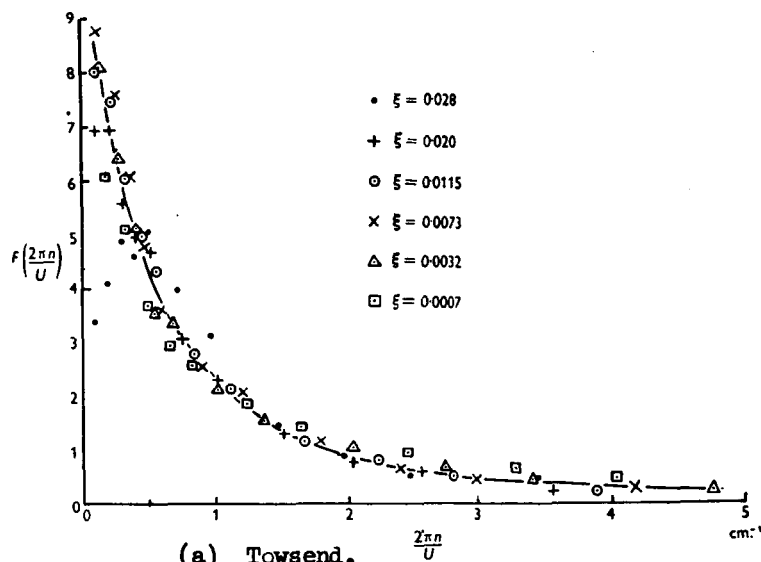


Figure 9.- Comparison of turbulence spectra in flat-plate boundary layer.
Presented as a function of distance from the wall.

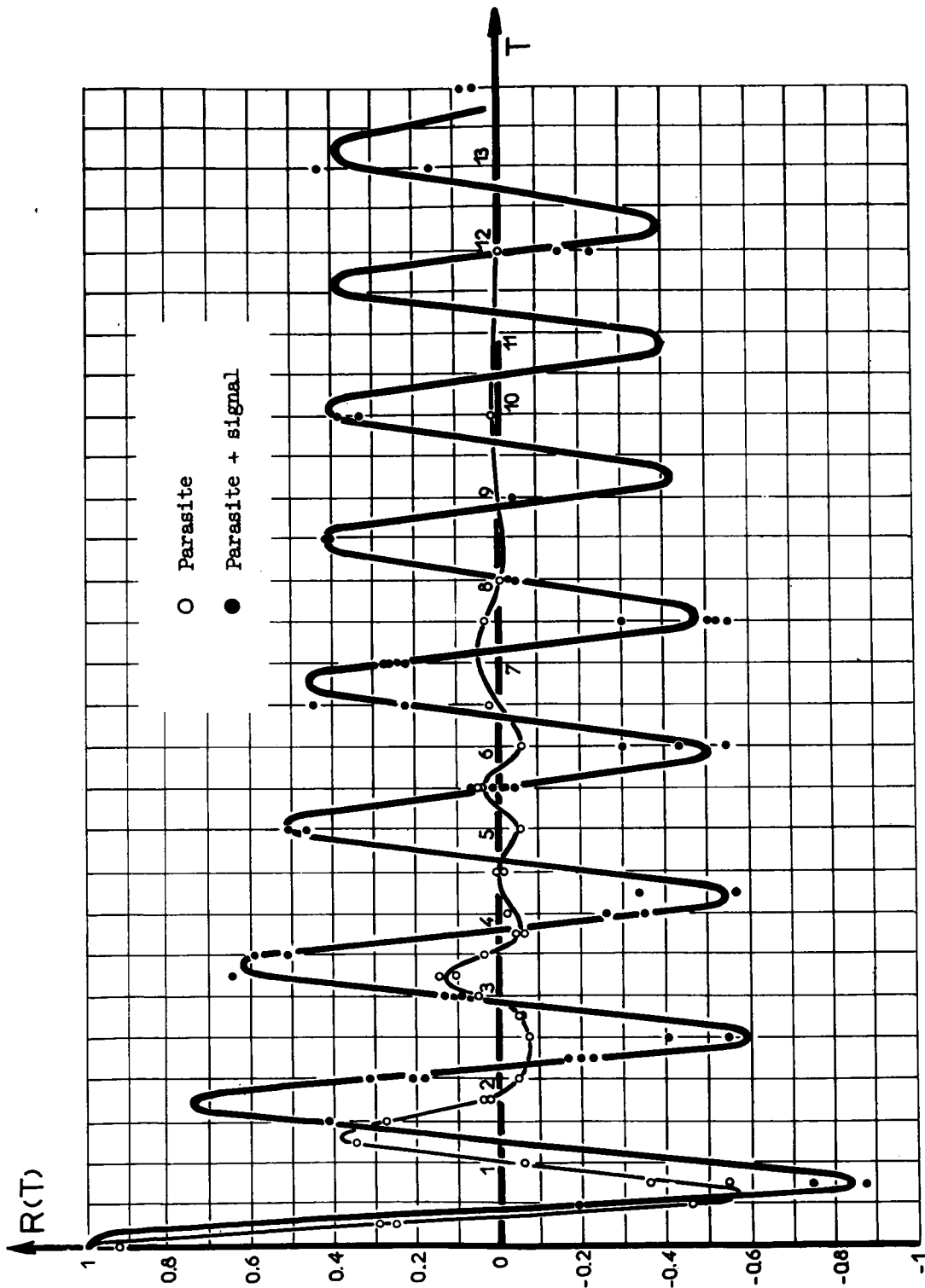


Figure 10.- Detection of a sine wave signal disturbed by a random parasite.